# AP PHYSICS C MECHANICS – SUMMER CHECKLIST

## REMIND INSTRUCTIONS

☐ The first document contains instructions for joining the Remind class discussion group. At any time after 7/15/20 and before the first day of school, please follow the instructions and join the group.

## SYLLABUS

☐ Document #2 is the course syllabus. Please read it carefully.

☐ You must have a three-ring binder and loose-leaf paper the first day of class.

☐ You must have a graphing calculator. I recommend either the TI-Nspire CX CAS or the TI-84+CE.

## STUDY NOTES

☐ Please read document #3 titled "AP Course Guidelines".

☐ Document #4 is an outline of the mathematical prerequisites required for AP Physics C. We will be using this mathematics throughout the course with little or no review beforehand. Please familiarize yourself with it.

## CONTRACT

☐ Please read and sign the contract (Document #5).

☐ Please have a parent or guardian read and sign the contract.

☐ Please bring the signed contract to the first day of class.

remind

# Sign up for important updates from Mr. Dominguez.

Get information for **Everglades High School** right on your phone—not on handouts.

Pick a way to receive messages for **AP Physics C Mechanics**:

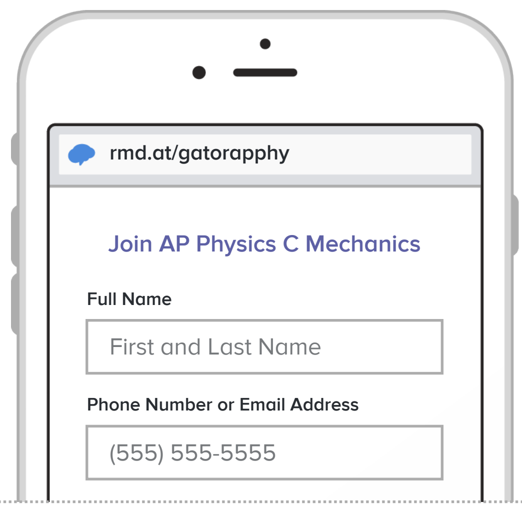
A

**If you have a smartphone, get push notifications.**

On your iPhone or Android phone, open your web browser and go to the following link:

rmd.at/gatorapphy

Follow the instructions to sign up for Remind. You'll be prompted to download the mobile app.

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**If you don't have a smartphone, get text notifications.**

Text the message @gatorapphy to the number 81010.

If you're having trouble with 81010, try texting @gatorapphy to (954) 883-9389.

** Standard text message rates apply.*


**Don't have a mobile phone?** Go to rmd.at/gatorapphy on a desktop computer to sign up for email notifications.

**Mr. Alberto Dominguez**
alberto.dominguez@browardschools.com

## AP PHYSICS C MECHANICS

**2021 AP Exam Date: Mon May 3, 12:00 noon**

### Course Description

AP Physics C Mechanics is equivalent to a one-semester, calculus-based, university-level physics course (i.e., PHY 2048 in Florida's Statewide Course Numbering System), especially appropriate for students planning to specialize or major in physical science or engineering. The course covers Newtonian mechanics.

### Laboratory Component

The course includes a hands-on laboratory component (20%) comparable to a semester-long introductory university-level physics laboratory course (i.e., PHY 2048L in Florida's Statewide Course Numbering System). Each student in the class will be required to maintain a portfolio of lab reports.

### Pre-requisite/Co-requisite

Students should have taken or be concurrently taking calculus. Basic differential and integral calculus are used throughout the course, starting on the first day.
Students should have completed an introductory physics course equivalent to Honors Physics with an overall grade of B or better.

### Required Materials

- Graphing calculator. I will be using a TI-84+CE in class, but any of the calculators on the AP approved list is acceptable.
- A three-ring binder to keep lecture notes, problem assignments and class handouts. It is essential that you have a binder with loose-leaf paper rather than a notebook because you will often have to pull sheets out for classwork or to turn in work.
- Loose-leaf paper, pens (black/blue for work and red/green for corrections), pencils, etc.
- Dry-erase markers

### Textbook

Serway & Jewett, *Physics for Scientists and Engineers 9th Edition* (Cengage, 2014)

### Grading

Daily homework assessments, quizzes upon completion of each topic (exact dates TBD), and tests upon completion of each of the seven units.

### Schedule of Topics (First Three Quarters)

Unit 1 – Kinematics [10 days]
Unit 2 – Newton's Laws of Motion [11 days]
Unit 3 – Work, Energy, Power [10 days]
Unit 4 – Systems of Particles and Linear Momentum [10 days]
Unit 5 – Rotation [10 days]
Unit 6 – Oscillations [6 days]
Unit 7 – Gravitation [6 days]

# Advanced Placement Course Guidelines

Students taking an Advanced Placement (AP) course at Everglades should understand the following information:

- An AP course is the equivalent of a college-level course. The curriculum for an AP course, as set by College Board, is designed to prepare students to take a national exam in May. Everglades students enrolled in an AP course are required to take the AP exam.

- Taking an AP course means having to complete assignments at a very fast pace. The pace of an AP course is set by College Board. To develop the skills necessary and to learn the content required for the AP exam, the course must move at a steady (some say rapid) pace. Students in an AP course must feel comfortable if the class moves on to another skill or new content before they have mastered the previous skills and content.

- Taking an AP course means doing more work. You can expect to have longer assignments and more work outside of regularly scheduled class time. Students should expect to study **at least** 90 minutes outside of class for every 90 minutes spent in class.

- The content and skills to be mastered in an AP course are more sophisticated than those in previous courses. Students moving into an AP course will generally see their grades go down. Almost every student in AP Calculus earned an A in Pre-Calculus Honors, but experience indicates that few will do so in AP Calculus. If you are a student for whom it is important to get an A in every course, then an AP class may not be right for you.

- Just because you are eligible to take an AP course does not mean you should take an AP course. Students who have a deep interest in the course material, who are willing to work longer hours, who can work independently and think abstractly in the discipline, AND who can balance such a commitment with their other obligations and goals (particularly other Everglades AP courses) tend to have the most positive experiences in an AP course.

# The Basics of Physics with Calculus

AP Physics C

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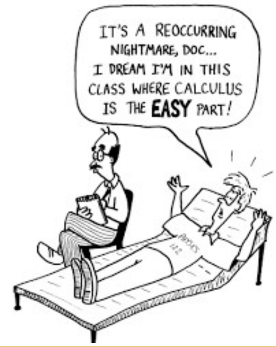


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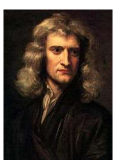
## Calculus

Isaac Newton and Gottfried Leibniz developed a sophisticated language of numbers and symbols called ***Calculus*** based on work. Newton began his work first but it was Leibniz who first published his findings. Both led the other towards accusations of plagiarism.



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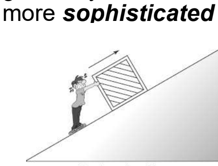
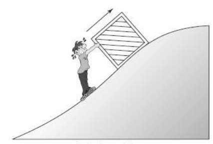
## What is calculus?

Calculus is simply very advanced algebra and geometry that has been tweaked to solve more ***sophisticated*** problems.



Regular math problem



Calculus problem

Question: How much energy does the man use to push the crate up the incline?

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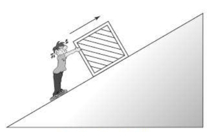
## The "regular" way

For the straight incline, the man pushes with an *unchanging* force, and the crate goes up the incline at an *unchanging* speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline.



Regular math problem

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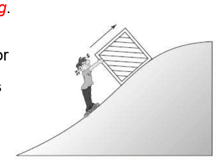
## The "calculus" way

For the curving incline, on the other hand, things are constantly *changing*. The steepness of the incline is *changing* — and not just in increments like it's one steepness for the first 10 feet then a different steepness for the next 10 feet — it's *constantly changing*. And the man pushes with a *constantly changing* force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also *changing*, not every second or every thousandth of a second, but *constantly changing* from one moment to the next. That's what makes it a calculus problem.



Calculus problem

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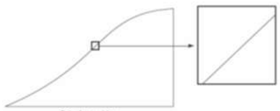
## What is calculus?

**It is a mathematical way to express something that is … CHANGING! It could be anything.**

**But here is the cool part:**




Calculus allows you to **ZOOM** in on a small part of the problem and apply the “regular” math tools.

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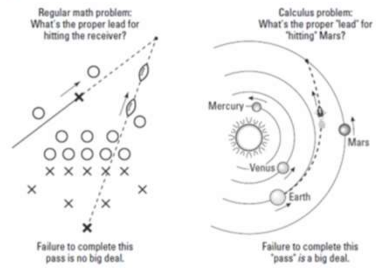
## “Regular” math vs. “Calculus”

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## Learn the lingo!

Calculus is about “rates of change”.

A ***RATE*** is anything divided by time.

***CHANGE*** is expressed by using the Greek letter, Delta, $\Delta$.

For example: Average ***SPEED*** is simply the “RATE at which DISTANCE changes”.

$$\bar{s} = \frac{\Delta d}{\Delta t}$$

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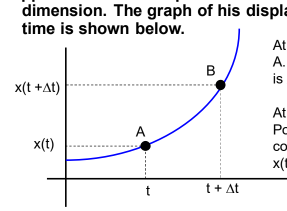
## The Derivative…aka….The SLOPE!

Since we are dealing with quantities that are changing it may be useful to define WHAT that change actually represents.

**Suppose an eccentric pet ant is constrained to move in one dimension. The graph of his displacement as a function of time is shown below.**




At time ***t***, the ant is located at Point A. While there, its position coordinate is x(t).

At time ***(t+ Δt)***, the ant is located at Point B. While there, its position coordinate is x(t + Δt)

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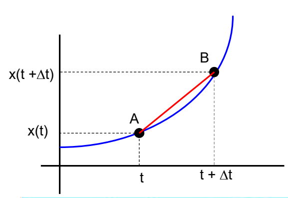
## The secant line and the slope

**Suppose a secant line is drawn between points A and B. Note: The slope of the secant line is equal to the rise over the run.**




$$Slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x(t+\Delta t) - x(t)}{(t+\Delta t) - t} = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

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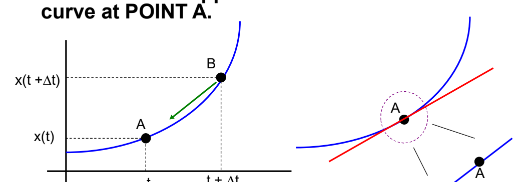
## The “Tangent” line

**READ THIS CAREFULLY!**

**If we hold POINT A fixed while allowing Δt to become very small. Point B approaches Point A and the secant approaches the TANGENT to the curve at POINT A.**




We are basically ZOOMING in at point A where upon inspection the line “APPEARS” straight. Thus the secant line becomes a **TANGENT LINE**.

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## The derivative

Mathematically, we just found the slope!

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{x(t+\Delta t) - x(t)}{\Delta t} = \text{slope of secant line}$$

$$\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \text{slope of tangent line}$$

**Lim stand for "LIMIT" and it shows the delta t approaches zero. As this happens the top numerator approaches a finite #.**

**This is what a derivative is. A derivative yields a NEW function that defines the rate of change of the original function with respect to one of its variables. In the above example we see, the rate of change of "x" with respect to time.**

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## The derivative

In most Physics books, the derivative is written like this:

$$\frac{d(x)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Mathematicians treat **dx/dt** as a **SINGLE SYMBOL** which means find the derivative. It is simply a mathematical operation.

**The bottom line: The derivative is the slope of the line tangent to a point on a curve.**

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## Derivative example

**Consider the function x(t) = 3t +2**

What is the time rate of change of the function? That is, what is the **NEW FUNCTION** that defines how ***x(t)*** changes as ***t*** changes.

This is actually very easy! The entire equation is linear and looks like y = mx + b . Thus we know from the beginning that the slope (the derivative) of this is equal to 3.

**Nevertheless: We will follow through by using the definition of the derivative**

$$x(t) = 3t + 2$$

$$\frac{d(x)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{[3(t+\Delta t)+2]-[3t+2]}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{3t + 3\Delta t + 2 - 3t - 2}{\Delta t} = \lim_{\Delta t \to 0} \frac{3\Delta t}{\Delta t} = 3$$

We didn't even need to INVOKE the limit because the delta t's cancel out.

Regardless, we see that we get a constant.

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## Example

**Consider the function x(t) = kt³, where k = proportionality constant equal to one in this case**..

$$x(t) = kt^3$$

$$\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{[(t+\Delta t)^3] - [t^3]}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{t^3 + 3t^2(\Delta t) + 3t(\Delta t)^2 + (\Delta t)^3 - t^3}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{3t^2(\Delta t) + 3t(\Delta t)^2 + (\Delta t)^3}{\Delta t}$$

$$= \lim_{\Delta t \to 0} [3t^2 + 3t(\Delta t) + (\Delta t^2)]$$

$$= 3t^2 \text{ \_or\_ } 3kt^2$$

**What happened to all the delta t's ? They went to ZERO when we invoked the limit!**

**What does this all mean?**

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## The MEANING?

$$\frac{d(kt^3)}{dt} = 3kt^2$$

**For example, if t = 2 seconds, using x(t) = kt³=(1)(2)³= 8 meters.**

**The derivative, however, tell us how our DISPLACEMENT (x) changes as a function of TIME (t). The rate at which Displacement changes is also called VELOCITY. Thus if we use our derivative we can find out how fast the object is traveling at t = 2 second. Since dx/dt = 3kt²=3(1)(2)²= 12 m/s**

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## THERE IS A PATTERN HERE!!!!

- Now if I had done the previous example with **kt²**, I would have gotten **2t¹**
- Now if I had done the above example with **kt⁴**, I would have gotten **4t³**
- Now if I had done the above example with **kt⁵**, I would have gotten **5t⁴**

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## Let's cheat…

$$If\_x(t) = kt^n, then: \quad \frac{dx}{dt} = nkt^{n-1}$$

$$If\_x(t) = 20t^3, then: \quad \frac{dx}{dt} = 60t^2$$

$$If\_f(x) = 5t^3 + 6t + 7, then: \quad \frac{dx}{dt} = 15t^2 + 6$$

$$If\_f(x) = 2t^6 + 7t^4 + 4t + 2, then: \quad \frac{dx}{dt} = 12t^5 + 28t^3 + 4$$

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## Common calculus derivative rules

$$-Ake^{-kt} = \frac{d(Ae^{-kt})}{dt}$$

$$k\cos(kt) = \frac{d[\sin(kt)]}{dt}$$

$$-k\sin(kt) = \frac{d[\cos(kt)]}{dt}$$

$$k(\frac{1}{t}) = \frac{d[\ln(kt)]}{dt}$$

$$\frac{k}{t} = kt^{-1} = \frac{d[kt^{-1}]}{dt} = (-1)kt^{-2} = \frac{-k}{t^2}$$

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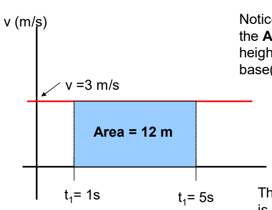
## The "AREA"

We have learned that the rate of change of displacement is defined as the VELOCITY of an object. Consider the graph below

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$




Notice that the 12 m happens to be the **AREA** under the line or the height( v = 3 m/s) times the base( t= 4 seconds) = 12 meters

This works really nice if the function is linear. What if it isn't?

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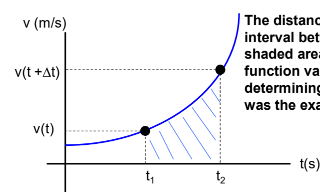
## The "Area"

**How do we determine HOW FAR something travels when the function is a curve?** Consider the velocity versus time graph below




**The distance traveled during the time interval between $t_1$ and $t_2$ equals the shaded area under the curve. As the function varies continuously, determining this area is NOT easy as was the example before.**

So how do we find the area?

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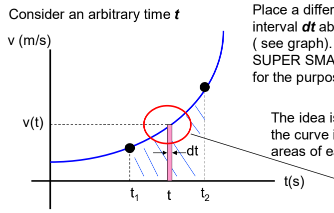
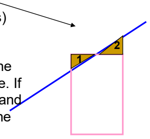
## Once again, we ZOOM in…

Consider an arbitrary time ***t***




Place a differential time interval ***dt*** about time ***t*** ( see graph). This rectangle is SUPER SMALL and is only visible for the purpose of an explanation.

The idea is that the AREA under the curve is the SUM of all the areas of each individual "***dt***".




With "dt" very small, area 1 fits into area 2 so that the approximate area is simply the area of the rectangle. If we find this area for ALL the small ***dt's*** between ***t1*** and ***t2***, then added them all up, we would end up with the TOTAL AREA or TOTAL DISPLACEMENT.

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## The "Integral"

The temptation is to use the conventional summation sign "Σ" . The problem is that you can only use the summation sign to denote the summing of **DISCRETE QUANTITIES** and **NOT** for something that is continuously varying. Thus, we cannot use it.

When a continuous function is summed, a different sign is used. It is called and **Integral**, and the symbol looks like this:

$$\int$$

When you are dealing with a situation where you have to integrate realize:

- **WE ARE GIVEN:** the derivative already
- **WE WANT:** The original function x(t)

So what are we basically doing? **WE ARE WORKING BACKWARDS!!!!! OR FINDING THE ANTI -DERIVATIVE**

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## Example

An object is moving at velocity with respect to time according to the equation v(t) = 2t.

a) What is the displacement function? Hint: What was the ORIGINAL FUCNTION BEFORE the "derivative? was taken?

$$x(t) = \int v\,dt \rightarrow \int (2t)dt =$$

$$x(t) = t^2$$

b) How FAR did it travel from t = 2s to t = 7s?

**These are your LIMITS!**

$$x(t) = \int_{t=2}^{t=7} v\,dt \rightarrow \int_{t=2}^{t=7} (2t)\,dt \rightarrow {}_{t=2}^{t=7} t^2$$

$$7^2 - 2^2 \rightarrow 49 - 4 = \textbf{45 m}$$

You might have noticed that in the above example we had to find the **change(Δ)** over the integral to find the area, that is why we subtract. This might sound confusing. But integration does mean SUM. What we are doing is finding the TOTAL AREA from 0-7 and then the TOTAL AREA from 0-2. Then we can subtract the two numbers to get JUST THE AREA from 2-7.

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## In summary…

So basically derivatives are used to find **SLOPES** and Integrals are used to find **AREAS**.

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

$$x = \int v\,dt \qquad v = \int a\,dt$$

**When do I use limits?**

There are only TWO things you will be asked to do.

- **DERIVE** – Simply find a function, which do not require limits
- **EVALUATE** – Find the function and solve using a given set of limits.

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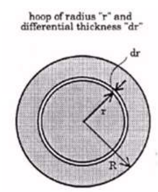
## Example




Here is a simple example of which you may be familiar with:

**Assume we know the circumference of a circle is 2πr, where *r* is the radius. How can we derive an expression for the area of a circle whose radius is *R*?**

**We begin by taking a differential HOOP of radius "*r*" and differential thickness "*dr*" as shown.**

If we determine the area of JUST OUR CHOSEN HOOP, we could do the calculation for ALL the possible hoops **inside** the circle.

Having done so, we would then SUM up all of those hoops to find the TOTAL AREA of the circle. The limits are going to be the two extremes, when r = R and when r = 0

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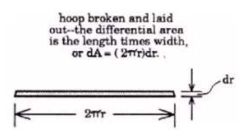
## Example cont'

If we break this hoop and make it flat, we see that it is basically a rectangle with the base equal to the circumference and the height equal to "dr".




$$Area = BasexHeight$$

$$dA = (2\pi r)dr$$

$$A = \int dA = \int_{r=0}^{R} (2\pi r)dr$$

$$= 2\pi \int_{r=0}^{R} (r)dr$$

$$= 2\pi [\frac{r^2}{2}]_{r=0}^{R}$$

$$= 2\pi [(\frac{R^2}{2}) - (0)]$$

$$= \pi R^2$$

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## Can we cheat? …YES!

Here is the integral equation. Simply take the exponent, add one, then divide by the exponent plus one.

$$\frac{x^{n+1}}{n+1}, n = \text{exponent}$$

$$3x^2 \rightarrow 3\int x^2 \rightarrow 3\left(\frac{x^{2+1}}{2+1}\right) = x^3$$

$$nx^{n-1} \rightarrow \frac{dx^3}{dt} \rightarrow 3x^2$$

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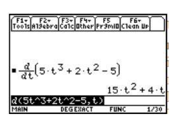
## The perfect tool…TI-89!

**The TI-89 graphing calculator can do ALL the calculus to truly need to do. Whether you are DERIVING or EVALUATING a function it can help you get the correct answer.**

First let me show you how to put a derivative into the calculator.



Using the SECOND key the derivative symbol is just above the number "8" key. A parenthesis will automatically appear. Type in the function you want like $5t^3+2t^2-5$ . After typing the function, put a comma after it and tell the calculator "WITH RESPECT TO WHAT" do you want to find the derivative of .

In this case, we want "WITH RESPECT TO TIME or t" . So we place a "t" after the comma and close the parenthesis. Hit enter and you will find the NEW FUNCTION = DERIVATIVE.

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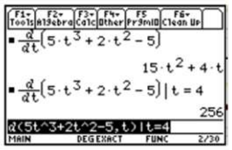
## The perfect tool…TI-89!




Suppose we want to now EVALUATE this function. In other words, I may want to what the velocity is at exactly t =4 seconds.

Velocity is the derivative of a displacement function! So we first have to derive the new function. Then we have to evaluate it at 4 seconds. All you do is enter a vertical line located next to the "7" key then type in t = 4. As you can see we get a velocity of 256 m/s. That is pretty fast!

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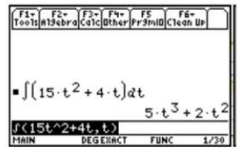
## The perfect tool…TI-89!




Start by using the second key. The integration symbol is located just ABOVE the "7". It, as well, will come with a parenthesis to begin with. Keep this in mind as you are entering in functions.

Enter the function, then use a comma, then state with respect to what. In this case we have TIME. Remember you can check your answer by taking the derivative of the function to see if you get the original equation.

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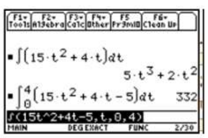
## The perfect tool…TI-89!




Now let's look at evaluating the function. THIS IS CALLED APPLYING THE LIMITS. In other words, over what period are we summing the area. It could be a length of time, a distance, an area, a volume...ANYTHING!

After stating what you are with respect to, enter in the LOWER LIMIT first, then the UPPER LIMIT, then close the parenthesis. So let's say this function was a VELOCITY function. The area under the graph represents DISPLACEMENT. So that means in FOUR SECONDS this object traveled 332 meters.

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## Example

**A particle moving in one dimension has a position function defined as:**

**$x(t) = 6t^4 - 2t$**

*a) At what point in time does the particle change its direction along the x-axis?*

**The body will change its direction when it reaches either its maximum or minimum x position. At that point it will reverse its direction. The velocity at the turn around point is ZERO. Thus the velocity function is:**

$$v = \frac{dx}{dt} = \frac{d(6t^4 - 2t)}{dt} =$$

$$v = 24t^3 - 2$$
$$0 = 24t^3 - 2$$
$$2 = 24t^3$$
$$t = \sqrt[3]{\frac{2}{24}} = 0.437s$$

34

## Example

*b) In what direction is the body traveling when its acceleration is 12 m/s²?*

***If we can determine the time at which a = +12 m/s, we can put that time back into our velocity function to determine the velocity of the body at that point of motion. Knowing the velocity (sign and all) will tell us the direction of motion.***

$$a = \frac{dv}{dt} = \frac{d(24t^3 - 2)}{dt}$$

$$a = 72t^2$$
$$12 = 72t^2$$
$$t = \sqrt{\frac{12}{72}} = 0.408s$$

$$v = 24t^3 - 2$$
$$v = 24(.408)^3 - 2$$
$$v = -0.37 m/s$$

**The velocity vector is negative, the body must be moving in the negative direction when the acceleration is +12 m/s/s.**

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# Vectors

AP Physics C

1

## Scalar

A **SCALAR** is ANY quantity in physics that has MAGNITUDE, but NOT a direction associated with it.

Magnitude – A numerical value with units.

| **Scalar Example** | **Magnitude** |
|---|---|
| Speed | 20 m/s |
| Distance | 10 m |
| Age | 15 years |
| Heat | 1000 calories |

2


3

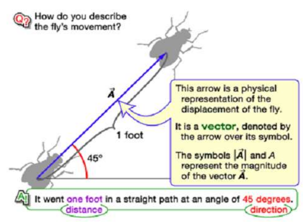
## Vector

A **VECTOR** is ANY quantity in physics that has BOTH MAGNITUDE and DIRECTION.

$$\vec{v}, \vec{x}, \vec{a}, \vec{F}$$

| **Vector** | **Magnitude & Direction** |
|---|---|
| Velocity | 20 m/s, N |
| Acceleration | 10 m/s/s, E |
| Force | 5 N, West |

Vectors are typically illustrated by drawing an ARROW above the symbol. The arrow is used to convey direction and magnitude.

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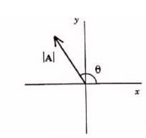
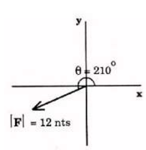
## Polar Notation




Polar notation defines a vector by designating the vector's magnitude |A| and angle θ relative to the +x axis. Using that notation the vector is written:

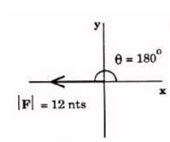
$$A = |A| \angle \theta$$



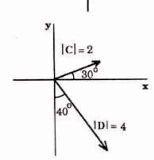

In this picture we have a force vector with magnitude 12 Newtons oriented at 210 degrees with the + x axis. It would be characterized as **F = 12 < 210**

5

## Polar Notation




In this picture we have a force vector of 12 Newtons oriented along the -x axis. However, polar notation is relative to the + x axis. Therefore, it would be characterized by **F = 12 < 180**




In this last picture we have 2 vectors. They are characterized by:

**C = 2 < 30**

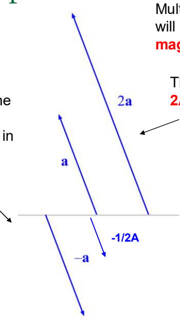
**D = 4 < - 50 or D = 4 < 310**

6

## Scalar Multiplication

Multiplying a vector by a scalar will ONLY CHANGE its **magnitude**.

Multiplying a vector by "-1" does not change the magnitude, but it does reverse it's direction or in a sense, it's angle.




Thus if **A = 12 < 105**, Then **2A = 24 < 105**

Thus if **A = 12 < 105**, then **-A = 12 < 285**

If **A = 12 < 105**, then **(-1/2)A = 6 < 285**

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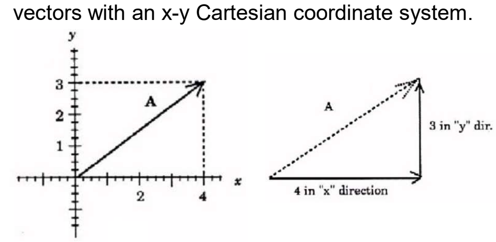
## Unit Vector Notation

An effective and popular system used in engineering is called **unit vector notation**. It is used to denote vectors with an x-y Cartesian coordinate system.

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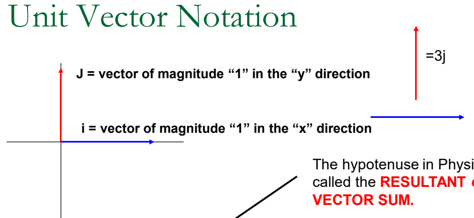
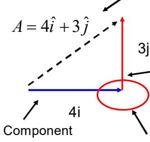
## Unit Vector Notation




The hypotenuse in Physics is called the **RESULTANT or VECTOR SUM.**

The LEGS of the triangle are called the **COMPONENTS**




Vertical Component

Horizontal Component

**NOTE:** When drawing a right triangle that conveys some type of motion, you MUST draw your components **HEAD TO TOE**.

9

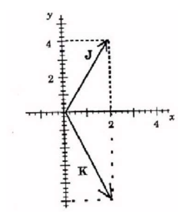
## Unit Vector Notation

$\hat{i}$ - unit vector = 1 in the + x direction

$\hat{j}$ - unit vector = 1 in the + y direction

$\hat{k}$ - unit vector = 1 in the + z direction

The proper terminology is to use the "hat" instead of the arrow. So we have i-hat, j-hat, and k-hat which are used to describe any type of motion in 3D space.




**How would you write vectors J and K in unit vector notation?**

$$J = 2\hat{i} + 4\hat{j}$$

$$K = 2\hat{i} - 5\hat{j}$$

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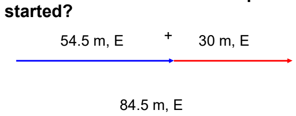
## Applications of Vectors

VECTOR ADDITION – If 2 similar vectors point in the SAME direction, add them.

- **Example: A man walks 54.5 meters east, then another 30 meters east. Calculate his displacement relative to where he started?**




Notice that the SIZE of the arrow conveys MAGNITUDE and the way it was drawn conveys DIRECTION.

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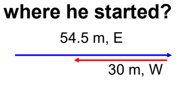
## Applications of Vectors

**VECTOR SUBTRACTION** - If 2 vectors are going in opposite directions, you **SUBTRACT.**

- **Example: A man walks 54.5 meters east, then 30 meters west. Calculate his displacement relative to where he started?**

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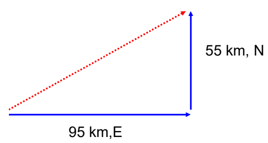
## Non-Collinear Vectors

When 2 vectors are perpendicular, you must use the Pythagorean theorem.

A man walks 95 km, East then 55 km, north. Calculate his RESULTANT DISPLACEMENT.




$$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$$
$$c = \text{Resultant} = \sqrt{95^2 + 55^2}$$
$$c = \sqrt{12050} = 109.8\ km$$

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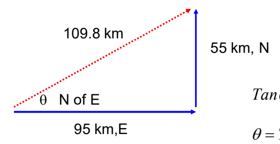
## BUT…..what about the VALUE of the angle???

Just putting North of East on the answer is NOT specific enough for the direction. We MUST find the VALUE of the angle.




To find the value of the angle we use a Trig function called TANGENT.

$$Tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{55}{95} = 0.5789$$
$$\theta = Tan^{-1}(0.5789) = 30^\circ$$

So the COMPLETE final answer is :

$$109.8km @ 30^\circ\ NofE$$
$$109.8 < 30$$
$$95\hat{i}\ m + 55\hat{j}\ m$$

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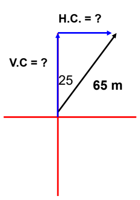
## What if you are missing a component?

Suppose a person walked 65 m, 25 degrees East of North. What were his horizontal and vertical components?




The goal: **ALWAYS MAKE A RIGHT TRIANGLE!**

**To solve for components, we often use the trig functions sine and cosine.**

$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$
$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = V.C. = 65\cos 25 = 58.91m, N\ or\ 58.91\hat{j}\ m$$
$$opp = H.C. = 65\sin 25 = 27.47m, E\ or\ 27.47\hat{i}\ m$$

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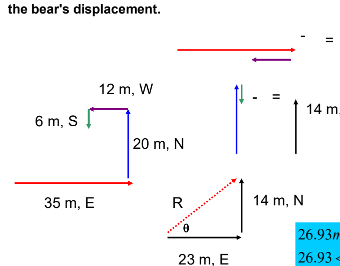
## Example

**A bear, searching for food wanders 35 meters east then 20 meters north. Frustrated, he wanders another 12 meters west then 6 meters south. Calculate the bear's displacement.**




$$R = \sqrt{14^2 + 23^2} = 26.93m$$
$$Tan\theta = \frac{14}{23} = .6087$$
$$\theta = Tan^{-1}(0.6087) = 31.3^\circ$$

The Final Answer:

$$26.93m @ 31.3^\circ\ NofE$$
$$26.93 < 31.3$$
$$23\hat{i}\ m + 14\hat{j}\ m$$

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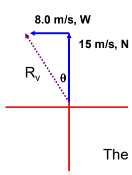
## Example

**A boat moves with a velocity of 15 m/s, N in a river which flows with a velocity of 8.0 m/s, west. Calculate the boat's resultant velocity with respect to due north.**




$$R_v = \sqrt{8^2 + 15^2} = 17\ m/s$$
$$Tan\theta = \frac{8}{15} = 0.5333$$
$$\theta = Tan^{-1}(0.5333) = 28.1^\circ$$

The Final Answer :

$$17\ m/s @ 28.1^\circ\ WofN$$
$$17\ m/s < 118.1^\circ$$
$$-8\hat{i}\ m/s + 15\hat{j}\ m/s$$

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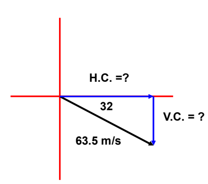
## Example

**A plane moves with a velocity of 63.5 m/s at 32 degrees South of East. Calculate the plane's horizontal and vertical velocity components.**




$$\text{cosine}\,\theta = \frac{adjacent\ side}{hypotenuse} \qquad \text{sine}\,\theta = \frac{opposite\ side}{hypotenuse}$$
$$adj = hyp\cos\theta \qquad opp = hyp\sin\theta$$

$$adj = H.C. = 63.5\cos 32 = 53.85\ m/s, E\ or\ 53.85\hat{i}$$
$$opp = V.C. = 63.5\sin 32 = 33.64\ m/s, S\ or\ -33.64\hat{j}$$

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## The "Dot" Product (Vector Multiplication)

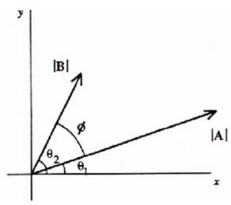
Multiplying 2 vectors sometimes gives you a SCALAR quantity which we call the **SCALAR DOT PRODUCT**.

In polar notation consider 2 vectors:
**A = |A| < $\theta_1$** & **B = |B| < $\theta_2$**

The dot product between A and B produces a SCALAR quantity. The magnitude of the scalar product is defined as:

$$A \bullet B = |A||B|\cos\phi$$

Where $\theta$ is the NET angle between the two vectors. As shown in the figure.

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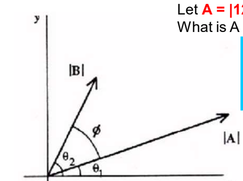
## The Scalar Dot Product

Let **A = |12| < 30**, Let **B = |5| < 65**
What is A "dot" B?

$$A \bullet B = |A||B|\cos\theta = |12||5|\cos 35$$
$$A \bullet B = 49.15$$




**In unit vector notation, it looks a little different. Consider:**

$$\mathbf{A} = \mathbf{A_x i} + \mathbf{A_y j} + \mathbf{A_z k}$$
$$\mathbf{B} = \mathbf{B_x i} + \mathbf{B_y j} + \mathbf{B_z k}$$

The "Dot" product between these is equal to:

$$A \bullet B = A_x B_x + A_y B_y + A_z B_z$$

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## The Scalar Dot Product

Example:

**Let A = (3i - 4j - 5k)**
**Let B = (2i + 7j + 3k)**

Therefore, A "dot" B = **(3)(2) + (-4)(7) +(-5)(3) = -37**

What is the **SIGNIFICANCE** of the dot product?

$$A \bullet B = |A||B|\cos\phi$$

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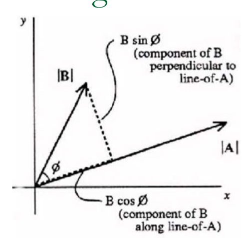
## The significance of the dot product




In this figure, vector B has been split into 2 components, one PARALLEL to vector A and one PERPENDICULAR to vector A. Notice that the component parallel to vector A has a magnitude of **|B|Cos θ**

THEREFORE when you find the DOT PRODUCT, the result is:
i) The MAGNITUDE of one vector, in this case |A| and,
ii) The MAGNITUDE of the 2nd vector's component that runs **parallel** to the first vector. (That is where the cosine comes from)

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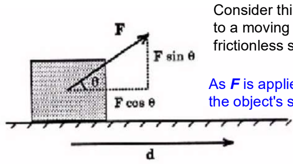
## Dot Products in Physics




Consider this situation: A force ***F*** is applied to a moving object as it transverses over a frictionless surface for a displacement, ***d***.

As ***F*** is applied to the object it will increase the object's speed!

But which part of ***F*** really causes the object to increase in speed?

It is **|F|Cos θ** ! Because it is **parallel** to the displacement **d**
In fact if you apply the dot product, you get **(|F|Cos θ)d,** which happens to be defined as "WORK" (check your equation sheet!)

$$A \bullet B = |A||B|\cos\theta$$
$$W = F \bullet x = |F||x|\cos\theta$$

Work is a type of energy and energy DOES NOT have a direction, that is why WORK is a scalar or in this case a SCALAR PRODUCT (AKA DOT PRODUCT).

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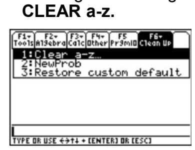
## Dot products using the TI-89

Here is where having the TI-89 is going to come if handy. You can actually enter vector into your calculator in UNIT VECTOR NOTATION. On your calculator we first need to clear any data that might be lurking in its memory. **Go to F6, then choose CLEAR a-z.**




This will wipe out any and all stored values that you have assigned to variable letters.

Now let's learn how to type a vector into the calculator in unit vector notation.

Suppose we had two vectors, one which represented the force and the other which represents the displacement as in our **WORK** example earlier.

$$\vec{F} = 3\hat{i} + 7\hat{j} - 2\hat{k}$$
$$\vec{d} = -5\hat{i} + 2\hat{j}$$

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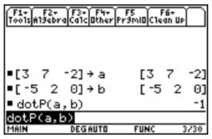
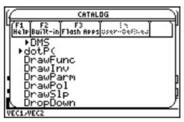
## Dot products using the TI-89

You start off by using brackets, then separate each number by a comma, then close the brackets. Then hit the STO button, then "a" as shown below in the diagram. Then do the same thing for the next vector except store this as "b"



$$\vec{F} = 3\hat{i} + 7\hat{j} - 2\hat{k}$$
$$\vec{d} = -5\hat{i} + 2\hat{j}$$



Now that you have assigned each vector a variable. On the calculator go to CATALOG and find dotP( from the menu). Hit enter. Then type in a,b and close the parenthesis as shown in the diagrams.

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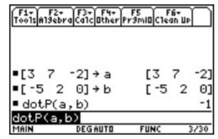
## Dot products using the TI-89



$$\vec{F} = 3\hat{i} + 7\hat{j} - 2\hat{k}$$
$$\vec{d} = -5\hat{i} + 2\hat{j}$$

We see we get "-1" for the WORK. What does this mean?

Remember, the dot product gives you a SCALAR which has NO DIRECTION. The negative here simply means that the Force and Displacement oppose each other yet are still PARALLEL.

**An example would be the WORK done by friction to slow a base runner down.**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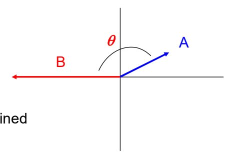
## The "Cross" Product (Vector Multiplication)

Multiplying 2 vectors sometimes gives you a VECTOR quantity which we call the **VECTOR CROSS PRODUCT**.

In polar notation consider 2 vectors:
**A = |A| < $\theta_1$ & B = |B| < $\theta_2$**




The cross product between A and B produces a VECTOR quantity. The magnitude of the vector product is defined as:

$$|AxB| = |A||B|\sin\phi$$

Where $\theta$ is the NET angle between the two vectors. As shown in the figure.

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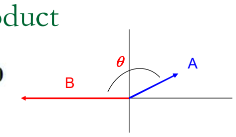
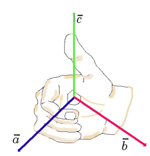
## The Vector Cross Product

Example:
**Let A = |5| < 30, Let B = |12| < 180**
**What is A "cross" B?**




$$A \otimes B = |A||B|\sin\theta = |12||5|\sin 150$$
$$A \otimes B = 30\hat{k}$$



What about the direction???? Positive k-hat??? We can use what is called the RIGHT HAND THUMB RULE.

- Fingers are the first vector, A
- Palm is the second vector, B
- **Thumb is the direction of the cross product.**
- Cross your fingers, A, towards, B so that they CURL. The direction it moves will be either clockwise (NEGATIVE) or counter clockwise (POSITIVE)

In our example, the thumb points OUTWARD which is the Z axis and thus our answer would be 30 k-hat since the curl moves counter clockwise.

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## Cross Products and Unit Vectors

Consider:
**A = $A_x$i + $A_y$j + $A_z$k**
**B = $B_x$i + $B_y$j + $B_z$k**

The cross product between B and A produces a VECTOR of which a 3x3 matrix is need to evaluate the magnitude and direction.

$$B \otimes A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix}$$

You start by making a 3x3 matrix with 3 columns, one for i, j, & k-hat. The components then go under each appropriate column.

Since B is the first vector it comes first in the matrix

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## Cross Products and Unit Vectors

$$B \otimes A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix}$$

You then make an X in the columns OTHER THAN the unit vectors you are working with.

- **For "i" , cross j x k**
- **For "j" , cross i x k**
- **For "k" , cross i x j**

Let's start with the i-hat vector: We cross j x k $\hat{i} = (B_y A_z) - (B_z A_y)$

Now the j-hat vector: We cross i x k $\hat{j} = (B_z A_x) - (B_x A_z)$

Now the k-hat vector: We cross i x j $\hat{k} = (B_x A_y) - (B_y A_x)$

30

## Example

Example: Let $\mathbf{A} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{B} = -2\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$. What is $\mathbf{B} \times \mathbf{A}$?

$$\mathbf{B} \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -4 & -6 \\ 3 & -4 & 5 \end{vmatrix}$$

Let's start with the i-hat vector: We cross j x k $\hat{i} = (-4)(5) - (-6)(-4) = -44$

Now the j-hat vector: We cross i x k $\hat{j} = (-6)(3) - (-2)(5) = -8$

Now the k-hat vector: We cross i x j $\hat{k} = (-2)(-4) - (-4)(3) = 20$

The final answer would be: $B \otimes A = -44\hat{i} - 8\hat{j} + 20\hat{k}$

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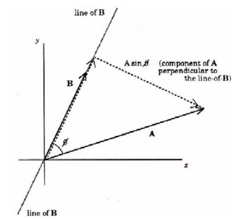
## The significance of the cross product

$$|AxB| = |A||B|\sin\phi$$




In this figure, vector A has been split into 2 components, one PARALLEL to vector B and one PERPENDICULAR to vector B. Notice that the component perpendicular to vector B has a magnitude of |A|sin θ

THEREFORE when you find the CROSS PRODUCT, the result is:
- i) The MAGNITUDE of one vector, in this case |B| and,
- ii) The MAGNITUDE of the 2nd vector's component that runs **perpendicular** to the first vector. ( that is where the sine comes from)

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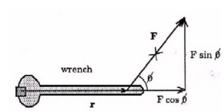
## Cross Products in Physics

There are many cross products in physics. You will see the matrix system when you learn to analyze circuits with multiple batteries. The cross product system will also be used in mechanics (rotation) as well as understanding the behavior of particles in magnetic fields.




A force ***F*** is applied to a wrench a displacement ***r*** from a specific point of rotation (ie. a bolt).

Common sense will tell us the larger ***r*** is the easier it will be to turn the bolt.

But which part of ***F*** actually causes the wrench to turn? *|F| Sin θ*

$$A \otimes B = |A||B|\sin\theta$$
$$\vec{F} \otimes \vec{r} = |F||r|\sin\theta$$

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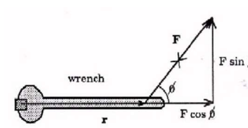
## Cross Products in Physics




$$A \otimes B = |A||B|\sin\theta$$
$$\vec{F} \otimes \vec{r} = |F||r|\sin\theta$$

**What about the DIRECTION?**

Which way will the wrench turn? **Counter Clockwise**

Is the turning direction positive or negative? **Positive**

Which way will the BOLT move? IN or OUT of the page? **OUT**

You have to remember that cross products give you a direction on the OTHER axis from the 2 you are crossing. So if "r" is on the x-axis and "F" is on the y-axis, the cross products direction is on the z-axis. In this case, a POSITIVE k-hat.

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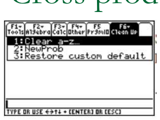
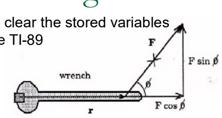
## Cross products using the TI-89

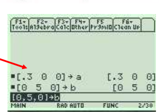



Let's clear the stored variables in the TI-89



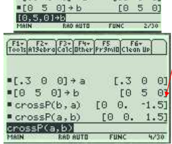

$A = r = 0.30m$
$B = F = 5N$
$B \otimes A = ?$

Since "r" is ONLY on the x-axis, it ONLY has an i-hat value. Enter, zeros for the other unit vectors.




Using **CATALOG**, find **crossP** from the menu.

Cross BxA, then AxB




Look at the answers carefully! Crossing BxA shows that we have a direction on the z-axis. Since it is negative it rotates CW. This means the BOLT is being tightened or moves IN to the page. Crossing AxB, causes the bolt to loosen as it moves OUT of the page

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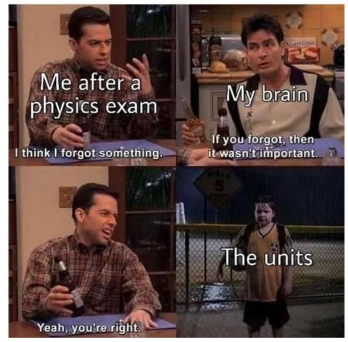
# Chapter 1:
# Physics and Measurement

1

## Units




2

## Standards of Length, Mass, and Time

Standards in measurement must:

- be readily accessible
- possess some property that can be measured reliably
- yield same result
- not change with time

3

## Length

**Length**: distance between two points in space

TABLE 1.1 Approximate Values of Some Measured Lengths

| | Length (m) |
|---|---|
| Distance from the Earth to the most remote known quasar | $2.7 \times 10^{26}$ |
| Distance from the Earth to the most remote normal galaxies | $3 \times 10^{26}$ |
| Distance from the Earth to the nearest large galaxy (Andromeda) | $2 \times 10^{22}$ |
| Distance from the Sun to the nearest star (Proxima Centauri) | $4 \times 10^{16}$ |
| One light-year | $9.46 \times 10^{15}$ |
| Mean orbit radius of the Earth about the Sun | $1.50 \times 10^{11}$ |
| Mean distance from the Earth to the Moon | $3.84 \times 10^{8}$ |
| Distance from the equator to the North Pole | $1.00 \times 10^{7}$ |
| Mean radius of the Earth | $6.37 \times 10^{6}$ |
| Typical altitude (above the surface) of a satellite orbiting the Earth | $2 \times 10^{5}$ |
| Length of a football field | $9.1 \times 10^{1}$ |
| Length of a housefly | $5 \times 10^{-3}$ |
| Size of smallest dust particles | $\sim 10^{-4}$ |
| Size of cells of most living organisms | $\sim 10^{-5}$ |
| Diameter of a hydrogen atom | $\sim 10^{-10}$ |
| Diameter of an atomic nucleus | $\sim 10^{-14}$ |
| Diameter of a proton | $\sim 10^{-15}$ |

**Meter: distance traveled by light in vacuum during a time interval of 1/299 792 458 second**

4

## Mass

TABLE 1.2 Approximate Masses of Various Objects

| | Mass (kg) |
|---|---|
| Observable Universe | $\sim 10^{52}$ |
| Milky Way galaxy | $\sim 10^{42}$ |
| Sun | $1.99 \times 10^{30}$ |
| Earth | $5.98 \times 10^{24}$ |
| Moon | $7.36 \times 10^{22}$ |
| Shark | $\sim 10^{3}$ |
| Human | $\sim 10^{2}$ |
| Frog | $\sim 10^{-1}$ |
| Mosquito | $\sim 10^{-5}$ |
| Bacterium | $\sim 1 \times 10^{-15}$ |
| Hydrogen atom | $1.67 \times 10^{-27}$ |
| Electron | $9.11 \times 10^{-31}$ |

a

Kilogram: **mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France**

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## Time

TABLE 1.3 Approximate Values of Some Time Intervals

| | Time Interval (s) |
|---|---|
| Age of the Universe | $4 \times 10^{17}$ |
| Age of the Earth | $1.3 \times 10^{17}$ |
| Average age of a college student | $6.3 \times 10^{8}$ |
| One year | $3.2 \times 10^{7}$ |
| One day | $8.6 \times 10^{4}$ |
| One class period | $3.0 \times 10^{3}$ |
| Time interval between normal heartbeats | $8 \times 10^{-1}$ |
| Period of audible sound waves | $\sim 10^{-3}$ |
| Period of typical radio waves | $\sim 10^{-6}$ |
| Period of vibration of an atom in a solid | $\sim 10^{-13}$ |
| Period of visible light waves | $\sim 10^{-15}$ |
| Duration of a nuclear collision | $\sim 10^{-22}$ |
| Time interval for light to cross a proton | $\sim 10^{-24}$ |


b

**Second: 9 192 631 770 times the period of vibration of radiation from the cesium-133 atom**

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## Pitfall Prevention 1.1

**Pitfall Prevention 1.1**

**Reasonable Values** Generating intuition about typical values of quantities when solving problems is important because you must think about your end result and determine if it seems reasonable. For example, if you are calculating the mass of a housefly and arrive at a value of 100 kg, this answer is *unreasonable* and there is an error somewhere.

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## Powers of 10

**TABLE 1.4** Prefixes for Powers of Ten

| Power | Prefix | Abbreviation | Power | Prefix | Abbreviation |
|---|---|---|---|---|---|
| $10^{-24}$ | yocto | y | $10^{3}$ | kilo | k |
| $10^{-21}$ | zepto | z | $10^{6}$ | mega | M |
| $10^{-18}$ | atto | a | $10^{9}$ | giga | G |
| $10^{-15}$ | femto | f | $10^{12}$ | tera | T |
| $10^{-12}$ | pico | p | $10^{15}$ | peta | P |
| $10^{-9}$ | nano | n | $10^{18}$ | exa | E |
| $10^{-6}$ | micro | $\mu$ | $10^{21}$ | zetta | Z |
| $10^{-3}$ | milli | m | $10^{24}$ | yotta | Y |
| $10^{-2}$ | centi | c | | | |
| $10^{-1}$ | deci | d | | | |

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## Modeling and Alternative Representations

A model is a simplified substitute for the real problem that allows us to solve the problem in a relatively simple way.

Two primary conditions for using particle model:

1. Size of the actual object of no consequence in analysis of its motion.
2. Any internal processes occurring in object of no consequence in analysis of its motion.

First category: **geometric model**

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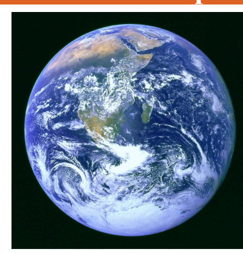
## Modeling and Alternative Representations



Second category: **simplification model**

Third category: **analysis model**

Fourth category: **structural model**

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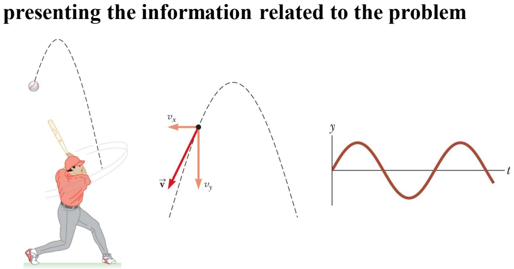
## Modeling and Alternative Representations

**A representation is a method of viewing or presenting the information related to the problem**

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## Dimensions

**TABLE 1.5** Dimensions and Units of Four Derived Quantities

| Quantity | Area ($A$) | Volume ($V$) | Speed ($v$) | Acceleration ($a$) |
|---|---|---|---|---|
| Dimensions | $L^2$ | $L^3$ | L/T | $L/T^2$ |
| SI units | $m^2$ | $m^3$ | m/s | $m/s^2$ |
| U.S. customary units | $ft^2$ | $ft^3$ | ft/s | $ft/s^2$ |

[3]The *dimensions* of a quantity will be symbolized by a capitalized, nonitalic letter such as L or T. The *algebraic symbol* for the quantity itself will be an italicized letter such as *L* for the length of an object or *t* for time.

speed: $[v] = \mathrm{L/T}$ area: $[A] = \mathrm{L}^2$

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## Dimensional Analysis

$$x = \frac{1}{2}at^2 \qquad \mathrm{L} = \frac{\mathrm{L}}{\cancel{\mathrm{T}^2}} \cdot \cancel{\mathrm{T}^2} = \mathrm{L}$$

$$x \propto a^n t^m$$

$$\left[a^n t^m\right] = \mathrm{L} = \mathrm{L}^1\mathrm{T}^0$$

$$\left(\mathrm{L}/\mathrm{T}^2\right)^n \mathrm{T}^m = \mathrm{L}^1\mathrm{T}^0 \rightarrow \left(\mathrm{L}^n\mathrm{T}^{m-2n}\right) = \mathrm{L}^1\mathrm{T}^0$$

$$x \propto at^2$$

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## Pitfall Prevention 1.2

**Pitfall Prevention 1.2**

**Symbols for Quantities** Some quantities have a small number of symbols that represent them. For example, the symbol for time is almost always $t$. Other quantities might have various symbols depending on the usage. Length may be described with symbols such as $x$, $y$, and $z$ (for position); $r$ (for radius); $a$, $b$, and $c$ (for the legs of a right triangle); $\ell$ (for the length of an object); $d$ (for a distance); $h$ (for a height); and so forth.

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## Example 1.1: Analysis of an Equation

Show that the expression $v = at$, where $v$ represents speed, $a$ acceleration, and $t$ an instant of time, is dimensionally correct.

$$[v] = \frac{\mathrm{L}}{\mathrm{T}}$$

$$[at] = \frac{\mathrm{L}}{\mathrm{T}^{\cancel{2}}}\cancel{\mathrm{T}} = \frac{\mathrm{L}}{\mathrm{T}}$$

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## Example 1.2: Analysis of a Power Law

Suppose we are told that the acceleration $a$ of a particle moving with uniform speed $v$ in a circle of radius $r$ is proportional to some power of $r$, say $r^n$, and some power of $v$, say $v^m$. Determine the values of $n$ and $m$ and write the simplest form of an equation for the acceleration.

$$a = kr^n v^m$$

$$\frac{\mathrm{L}}{\mathrm{T}^2} = \mathrm{L}^n\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^m = \frac{\mathrm{L}^{n+m}}{\mathrm{T}^m}$$

$$n + m = 1 \text{ and } m = 2 \qquad \rightarrow n = -1$$

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

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## Conversion of Units

$$1 \text{ mi} = 1\,609 \text{ m} = 1.609 \text{ km}$$
$$1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$
$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft}$$
$$1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

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## Conversion of Units

Convert 15.0 in. to centimeters.

$$1 \text{ in.} = 2.54 \text{ cm} \rightarrow \frac{2.54 \text{ cm}}{1 \text{ in.}}$$

$$15.0 \text{ in.} = (15.0\,\cancel{\text{in}}.)\left(\frac{2.54 \text{ cm}}{1\,\cancel{\text{in}}.}\right) = 38.1 \text{ cm}$$

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## Pitfall Prevention 1.3

**Pitfall Prevention 1.3**

**Always Include Units** When performing calculations with numerical values, include the units for every quantity and carry the units through the entire calculation. Avoid the temptation to drop the units early and then attach the expected units once you have an answer. By including the units in every step, you can detect errors if the units for the answer turn out to be incorrect.

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## Example 1.3: Is He Speeding?

On an interstate highway in a rural region of Wyoming, a car is traveling at a speed of 38.0 m/s. Is the driver exceeding the speed limit of 75.0 mi/h?

$$\left(38.0\ \cancel{\text{m}}/\cancel{\text{s}}\right)\left(\frac{1\ \text{mi}}{1609\ \cancel{\text{m}}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\,\text{h}}\right)=85.0\ \text{mi/h}$$

Yes, he is speeding!

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## Example 1.3: Is He Speeding?

What if the driver were from outside the United States and is familiar with speeds measured in kilometers per hour? What is the speed of the car in km/h?

$$\left(85.0\ \frac{\cancel{\text{mi}}}{\text{h}}\right)\left(\frac{1.609\ \text{km}}{1\ \cancel{\text{mi}}}\right)=137\ \text{km/h}$$

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## Example 1.4: Breaths in a Lifetime

Estimate the number of breaths taken during an average human lifetime.

$$1\ \cancel{\text{yr}}\left(\frac{400\ \cancel{\text{days}}}{1\ \cancel{\text{yr}}}\right)\left(\frac{25\ \cancel{\text{h}}}{1\ \cancel{\text{day}}}\right)\left(\frac{60\ \text{min}}{1\,\cancel{\text{h}}}\right)=6\times10^5\ \text{min}$$

$$\text{number of minutes}=(70\ \text{yr})(6\times10^5\ \text{min/yr})$$
$$=4\times10^7\ \text{min}$$

$$\text{number of breaths}=(10\ \text{breaths/min})(4\times10^7\ \text{min})$$
$$=4\times10^8\ \text{breaths}$$

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## Example 1.4: Breaths in a Lifetime

What if the average lifetime were estimated as 80 years instead of 70? Would that change our final estimate?

$$(80\ \text{yr})(6\times10^5\ \text{min/yr})=5\times10^7\ \text{min}$$

$$(10\ \text{breaths/min})(5\times10^7\ \text{min})=5\times10^8\ \text{breaths}$$

$$\text{on the order of }10^9\ \text{breaths}$$

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## Significant Figures


$$(6.0\pm0.1)\ \text{cm}$$

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## Significant Figures

$$1500 \text{ g} \rightarrow ? \text{ significant figures}$$

$$1.500\times10^3 \text{ g} \rightarrow 4 \text{ significant figures}$$
$$1.50\times10^3 \text{ g} \rightarrow 3 \text{ significant figures}$$
$$1.5\times10^2 \text{ g} \rightarrow 2 \text{ significant figures}$$

$$2.3\times10^{-4} \rightarrow 2 \text{ significant figures} \rightarrow 0.00023$$
$$2.30\times10^{-4} \rightarrow 3 \text{ significant figures} \rightarrow 0.000230$$

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## Significant Figures

When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures. The same rule applies to division.

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## Significant Figures

$$A = \pi r^2 = \pi(6.0 \text{ cm})^2 = \boxed{1.1\times10^2 \text{ cm}^2}$$

calculator answer: 113.0973355

~~113 cm~~

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## Significant Figures

When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum or difference.

$$23.2 + 5.174 = 28.374$$
$$\rightarrow 23.2 \text{ has one decimal place} \rightarrow \text{sum} = 28.4$$

$$1.0001 + 0.0003 = 1.0004$$
$$1.002 - 0.998 = 0.004$$

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## Significant Figures

In this book, most of the numerical examples and end-of-chapter problems will yield answers having three significant figures. When carrying out estimation calculations, we shall typically work with a single significant figure.

last digit dropped > 5: increase last retained digit by 1: 1.356 → 1.35

last digit dropped = 5: increase last retained rounded to nearest even number: 1.345 → 1.34

last digit dropped < 5: leave last retained as is: 1.343 → 1.34

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## Pitfall Prevention 1.4

**Pitfall Prevention 1.4**

**Read Carefully** Notice that the rule for addition and subtraction is different from that for multiplication and division. For addition and subtraction, the important consideration is the number of *decimal places*, not the number of *significant figures*.

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## Example 1.5: Installing a Carpet

A carpet is to be installed in a rectangular room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m. Find the area of the room.

$$12.71 \text{ m} \times 3.46 \text{ m} = 43.9766 \text{ m}^2$$

$$3.46 \rightarrow 3 \text{ sig figs} \rightarrow A = 44.0 \text{ m}^2$$

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## Pitfall Prevention 1.5

**Pitfall Prevention 1.5**

**Symbolic Solutions** When solving problems, it is very useful to perform the solution completely in algebraic form and wait until the very end to enter numerical values into the final symbolic expression. This method will save many calculator keystrokes, especially if some quantities cancel so that you never have to enter their values into your calculator! In addition, you will only need to round once, on the final result.

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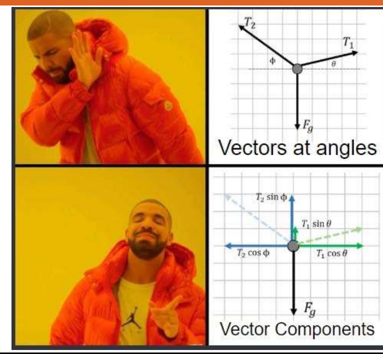
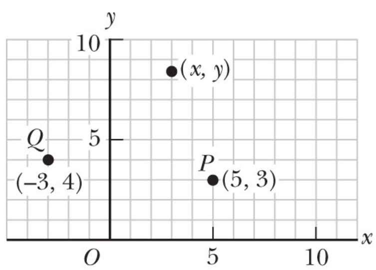
# Chapter 3: Vectors


1




2

## Cartesian Coordinate System

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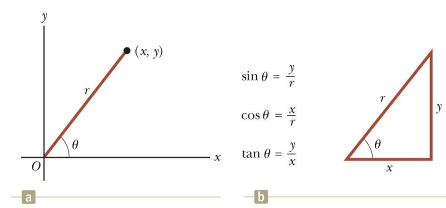
## Polar Coordinate System




$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$
$$r = \sqrt{x^2 + y^2}$$

4

## Vectors

0304_TheBasicsOfTwoDimensionalVectors.cdf

5

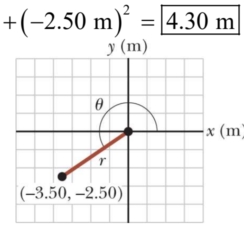
## Example 3.1: Polar Coordinates

The Cartesian coordinates of a point in the *xy* plane are $(x, y) = (-3.50, -2.50)$ m as shown in the figure. Find the polar coordinates of this point.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = \boxed{4.30\text{ m}}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216°$$




6

## Vector and Scalar Quantities

A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.

A **vector quantity** is completely specified by a number with an appropriate unit (the *magnitude* of the vector) plus a direction.

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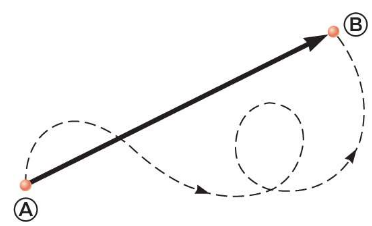
## Displacement Vector

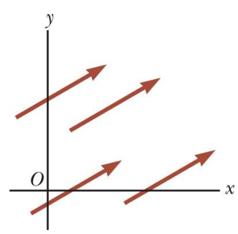



8

## Basic Vector Arithmetic

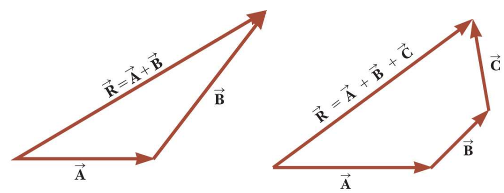
$\vec{\mathbf{A}} = \vec{\mathbf{B}}$ only if $A = B$

Both point in the same direction along parallel lines




9

## Vector Addition




$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$ (**commutative law of addition**)

$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}$ (**associative law of addition**)

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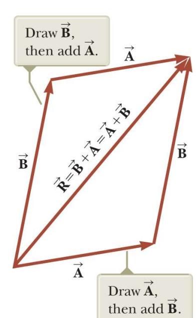
## Vector Addition is Commutative

0303_VectorAdditionIsCommutative.cdf

11

## Vector Addition




12

## Vector Addition

0303_SumOfTwoVectors.cdf

0303_2DVectorAddition.cdf

0303_HeadToToeVectorAddition.cdf

13

## Pitfall Prevention 3.1

**Pitfall Prevention 3.1**

**Vector Addition Versus Scalar Addition** Notice that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$ is very different from $A + B = C$. The first equation is a vector sum, which must be handled carefully, such as with the graphical method. The second equation is a simple algebraic addition of numbers that is handled with the normal rules of arithmetic.

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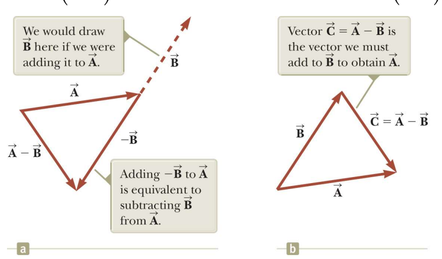
## Vector Subtraction and Scalar Multiplication

$\vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$

$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$




scalar multiplication: $m\vec{\mathbf{A}}$

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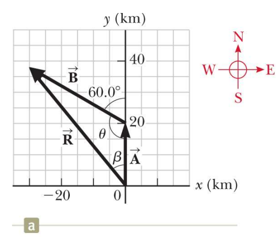
## Example 3.2: A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in the figure. Find the magnitude and direction of the car's resultant displacement.

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## Example 3.2: A Vacation Trip

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$R = \sqrt{(20.2\text{ km})^2 + (35.0\text{ km})^2 - 2(20.2\text{ km})(35.0\text{ km})\cos 120°}$$

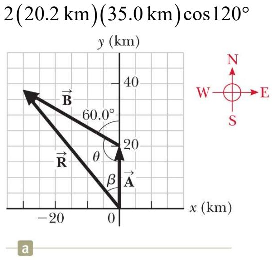
$$= \boxed{48.2\text{ km}}$$

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta$$

$$= \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = \boxed{38.9°}$$

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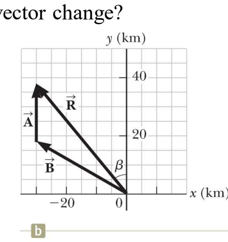
## Example 3.2: A Vacation Trip

Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

They would not change.

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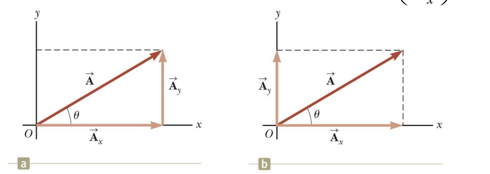
## Components of a Vector

$$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta \qquad A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

$$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



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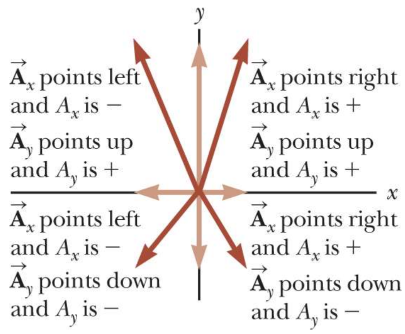
## Components of a Vector

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## Pitfall Prevention 3.2

**Pitfall Prevention 3.2**

***x* and *y* Components** Equations 3.8 and 3.9 associate the cosine of the angle with the *x* component and the sine of the angle with the *y* component. This association is true *only* because we measured the angle $\theta$ with respect to the *x* axis, so do not memorize these equations. If $\theta$ is measured with respect to the *y* axis (as in some problems), these equations will be incorrect. Think about which side of the triangle containing the components is adjacent to the angle and which side is opposite and then assign the cosine and sine accordingly.

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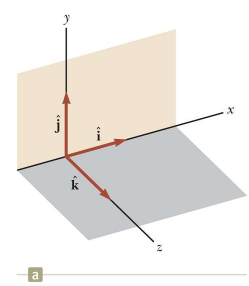
## Unit Vectors

$\hat{\mathbf{i}}$

$\hat{\mathbf{j}}$

$\hat{\mathbf{k}}$



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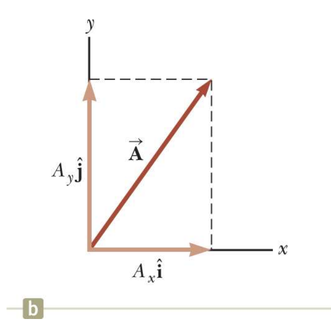
## Components of a Vector and Unit Vectors



$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$

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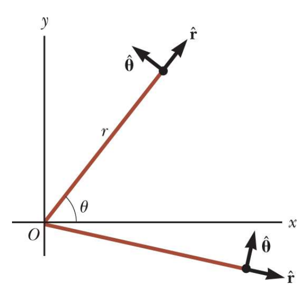
## Components of a Vector and Unit Vectors



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## Vector Addition using Components

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} \Rightarrow$$

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

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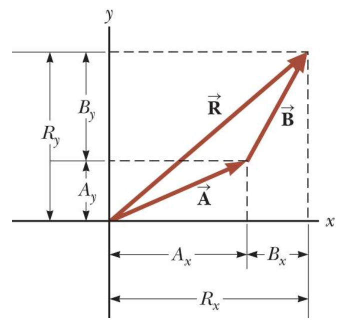
## Vector Addition using Components

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## Magnitude of a Vector

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{\left(A_x + B_x\right)^2 + \left(A_y + B_y\right)^2}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$$

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## Addition of N Vectors

0303_AdditionOfNVectorsIn2D.cdf

28

## Pitfall Prevention 3.3

**Pitfall Prevention 3.3**

**Tangents on Calculators** Equation 3.17 involves the calculation of an angle by means of a tangent function. Generally, the inverse tangent function on calculators provides an angle between −90° and +90°. As a consequence, if the vector you are studying lies in the second or third quadrant, the angle measured from the positive *x* axis will be the angle your calculator returns plus 180°.

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## Vectors in Three Dimensions

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

$$R_z = A_z + B_z$$

$$R = \sqrt{R_x^{\ 2} + R_y^{\ 2} + R_z^{\ 2}}$$

$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

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## Example 3.3: The Sum of Two Vectors

Find the sum of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the *xy* plane and given by

$$\vec{\mathbf{A}} = \left(2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}\right) \text{ and } \vec{\mathbf{B}} = \left(2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A_x = 2.0,\ A_y = 2.0,\ A_z = 0$$

$$B_x = 2.0,\ B_y = -4.0,\ B_z = 0$$

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## Example 3.3: The Sum of Two Vectors

$$\begin{aligned}\vec{\mathbf{R}} &= \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} \\ &= \left(2.0 + 2.0\right)\hat{\mathbf{i}} + \left(2.0 - 4.0\right)\hat{\mathbf{j}} \\ &= 4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}}\end{aligned}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0)^2 + (2.0)^2} = \sqrt{20} = \boxed{4.5}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0}{4.0} = -0.50 \rightarrow \theta = 333°$$

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## Example 3.4: The Resultant Displacement

A particle undergoes three consecutive displacements: $\Delta\mathbf{r}_1 = \left(15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}}\right)$cm, $\Delta\mathbf{r}_2 = \left(23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}}\right)$cm, and $\Delta\mathbf{r}_3 = \left(-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}}\right)$cm. Find unit-vector notation for the resultant displacement and magnitude.

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## Example 3.4: The Resultant Displacement

$$\begin{aligned}\Delta\vec{\mathbf{r}} &= \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3 \\ &= (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm} \\ &\quad + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm} \\ &= \left(25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}}\right)\text{cm}\end{aligned}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = \boxed{40\text{ cm}}$$

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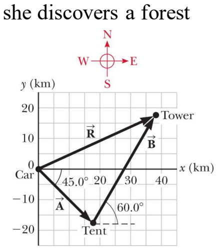
## Example 3.5: Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

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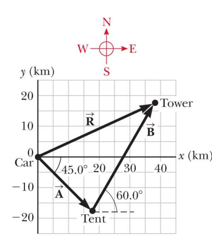
## Example 3.5: Taking a Hike

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = \boxed{17.7\text{ km}}$$

$$A_y = A\sin(-45.0°) = (25.0\text{ km})(-0.707) = \boxed{-17.7\text{ km}}$$

$$B_x = A\cos(60.0°) = (40.0\text{ km})(0.500) = \boxed{20.0\text{ km}}$$

$$B_y = A\sin(60.0°) = (40.0\text{ km})(0.866) = \boxed{34.6\text{ km}}$$

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## Example 3.5: Taking a Hike

(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = \boxed{37.3\text{ km}}$$

$$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km} = \boxed{17.0\text{ km}}$$

$$\vec{\mathbf{R}} = \left(37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}}\right)\text{ km}$$

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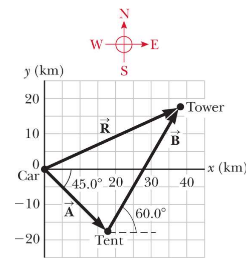
## Example 3.5: Taking a Hike

$$\vec{\mathbf{R}} = \left(37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}}\right)\text{ km}$$

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## Example 3.5: Taking a Hike

After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

$$\vec{\mathbf{R}}_{\text{car}} = -\vec{\mathbf{R}} = \left(-37.7\hat{\mathbf{i}} - 17.0\hat{\mathbf{j}}\right)\text{ km}$$

$$\tan\theta = \frac{R_{\text{car},y}}{R_{\text{car},x}} = \frac{-17.0\text{ km}}{-37.7\text{ km}} = 0.450$$

$$\Rightarrow \theta = 204.2°,\text{ or } 24.2° \text{ south of west}$$

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## Section 7.3: The Scalar (Dot) Product Of Two Vectors

40

## The Scalar Product

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\cos\theta$$

$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

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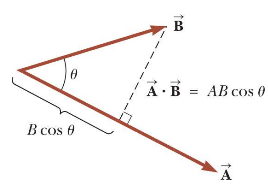
## The Scalar Product




$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\cos\theta$$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$

$$\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$$

$$\vec{\mathbf{A}} \perp \vec{\mathbf{B}} \quad \left(\theta = 90°\right) \Rightarrow \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$$

$$\vec{\mathbf{A}} \square \vec{\mathbf{B}} \quad \left(\theta = 0°\right) \Rightarrow \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB$$

$$\vec{\mathbf{A}} \square \vec{\mathbf{B}} \quad \left(\theta = 180°\right) \Rightarrow \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = -AB$$

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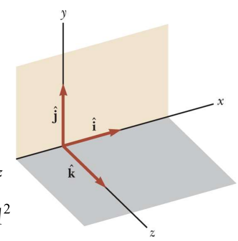
## The Scalar Product

$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}=1 \qquad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}=0$

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$
$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$




$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$
$$\vec{\mathbf{A}}\cdot\vec{\mathbf{A}} = A_x^2 + A_y^2 + A_z^2 = A^2$$

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## The Scalar Product

0703_DotProduct.cdf

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## Example 7.2: The Scalar Product

The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given by

$$\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} \text{ and } \vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

(A) Determine the scalar product $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}$.

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\cdot\left(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)$$
$$= -2\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\cdot 2\hat{\mathbf{j}}$$
$$= 2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = \boxed{4}$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x + A_yB_y = (2)(-1) + (3)(2) = 4$$

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## Example 7.2: The Scalar Product

(B) Find the angle $\theta$ between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$
$$\mathbf{B} = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3°$$

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**Section 11.1:**
**The Vector (Cross) Product**
**Of Two Vectors**

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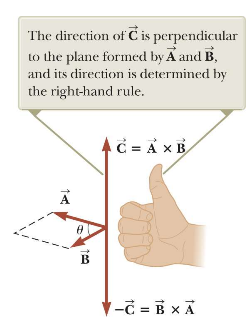
## The Vector Product

$$\vec{\mathbf{C}} = \vec{\mathbf{A}}\times\vec{\mathbf{B}}$$

$$C = AB\sin\theta$$

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## The Vector Product

Properties of vector product:

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = -\vec{\mathbf{B}}\times\vec{\mathbf{A}}$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = 0 \ \left(\vec{\mathbf{A}}\,\square\,\vec{\mathbf{B}}\right) \qquad \Rightarrow \vec{\mathbf{A}}\times\vec{\mathbf{A}} = 0$$

If $\vec{\mathbf{A}}\perp\vec{\mathbf{B}}$, then $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}+\vec{\mathbf{C}}\right) = \vec{\mathbf{A}}\times\vec{\mathbf{B}}+\vec{\mathbf{A}}\times\vec{\mathbf{C}}$$

$$\frac{d}{dt}(\vec{\mathbf{A}}\times\vec{\mathbf{B}}) = \frac{d\vec{\mathbf{A}}}{dt}\times\vec{\mathbf{B}} + \vec{\mathbf{A}}\times\frac{d\vec{\mathbf{B}}}{dt}$$

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## The Vector Product

$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = -\hat{\mathbf{j}}\times\hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}}\times\hat{\mathbf{k}} = -\hat{\mathbf{k}}\times\hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}}\times\hat{\mathbf{i}} = -\hat{\mathbf{i}}\times\hat{\mathbf{k}} = \hat{\mathbf{j}}$$

$$\vec{\mathbf{A}}\times\left(-\vec{\mathbf{B}}\right) = -\vec{\mathbf{A}}\times\vec{\mathbf{B}}$$

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## The Vector Product

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}\hat{\mathbf{i}} + \begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix}\hat{\mathbf{j}} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}\hat{\mathbf{k}}$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \left(A_yB_z - A_zB_y\right)\hat{\mathbf{i}} + \left(A_zB_x - A_xB_z\right)\hat{\mathbf{j}} + \left(A_xB_y - A_yB_x\right)\hat{\mathbf{k}}$$

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## The Vector Product

1101_CrossProduct.cdf

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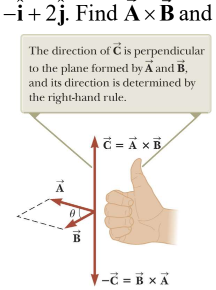
## Example 11.1: The Vector Product

Two vectors lying in the $xy$ plane are given by the equations $\vec{\mathbf{A}} = 2\hat{\mathbf{i}}+3\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}}+2\hat{\mathbf{j}}$. Find $\vec{\mathbf{A}}\times\vec{\mathbf{B}}$ and verify that $\vec{\mathbf{A}}\times\vec{\mathbf{B}} = -\vec{\mathbf{B}}\times\vec{\mathbf{A}}$.

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## Example 11.1: The Vector Product

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)\times\left(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = 2\hat{\mathbf{i}}\times\left(-\hat{\mathbf{i}}\right)+2\hat{\mathbf{i}}\times2\hat{\mathbf{j}}+3\hat{\mathbf{j}}\times\left(-\hat{\mathbf{i}}\right)+3\hat{\mathbf{j}}\times2\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = 0+4\hat{\mathbf{k}}+3\hat{\mathbf{k}}+0 = \boxed{7\hat{\mathbf{k}}}$$

$$\vec{\mathbf{B}}\times\vec{\mathbf{A}} = \left(-\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)\times\left(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{B}}\times\vec{\mathbf{A}} = \left(-\hat{\mathbf{i}}\right)\times2\hat{\mathbf{j}}+\left(-\hat{\mathbf{i}}\right)\times3\hat{\mathbf{j}}+2\hat{\mathbf{j}}\times2\hat{\mathbf{i}}+2\hat{\mathbf{j}}\times3\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}}\times\vec{\mathbf{A}} = 0-3\hat{\mathbf{k}}-4\hat{\mathbf{k}}+0 = -7\hat{\mathbf{k}}$$

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# Advanced Placement Contract – Physics C Mechanics

**Requirements:**

- Before the first day of class, students should read and agree to abide by the conditions stated in the AP Course Guidelines, the course syllabus, and this contract.
- Some homework assignments will not be turned in. Students must understand that this does NOT mean there is no homework. The learning objective for such assignments is for the student to understand how to solve the suggested problems. If the student does not complete the homework, the student will most likely fail the corresponding quiz.
- Assignments that do have to be turned in should be completed by the due dates given. Other than in the case of absence, late work will not be accepted.
- Students should be prepared for class every day, ready to participate fully in class discussions, individual class work, quizzes and/or examinations, and any other work determined necessary by the teacher.
- Students are required to take the first semester exam. There are no exemptions available for the first semester exam.
- Exam solution sessions (particularly for 4th quarter mock exams) will be scheduled immediately after school on certain dates (advance notice will be given). Students are responsible for their own transportation to classes scheduled when buses do not normally run. Material covered during these sessions will be essential knowledge; these are not optional.
- Cheating will not be tolerated in any form. Students who cheat will receive a grade of zero on the assignment or assessment and will be subject to disciplinary action.
- Students are required to take the AP exam on Monday, May 3, 2021, 12:00 noon – 2:00 pm (college credit may be given for students who earn a score of 3 or better, decided by individual colleges at their sole discretion). Students who do not take the AP exam will not get the additional quality point in their GPA calculation, will not be eligible for exemption from the second semester exam, and will incur a financial obligation.

I affirm that I have read this contract and agree to abide by its stipulations.

________________________________ ________________
Student Signature Date

I affirm that I have read this contract and support my student's decision to take this course. I understand the ramifications of this course selection.

________________________________ ________________
Parent/Guardian Signature Date

Please return this contract to Mr. Dominguez on the first day of class.